

## Accelerated Life Test Plots



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### Summary

The **Accelerated Life Test Plots** procedure is designed to plot data from an accelerated life test in which failure times have been recorded and the mean time to failure (MTTF) or a selected percentile has been estimated at a number of different levels of an accelerating variable. The MTTF or percentile  $P$  is assumed to follow one of several commonly used acceleration models. Given estimates at two or more levels of the accelerating variable, the procedure will fit the selected model and extrapolate the MTTF or percentile to a normal operating level.

**Sample StatFolio:** *altplots.sgp*

### Sample Data:

The file *circuits.sgd* contains data from an accelerated life test, where failures were caused by a chemical reaction within an integrated circuit. 10 items were tested at each of 5 different junction temperatures and the failure times recorded. Testing of each item stopped after 3,000 hours if the unit had not yet failed. A portion of the data, reported by Meeker and Escobar (1998), is shown below:

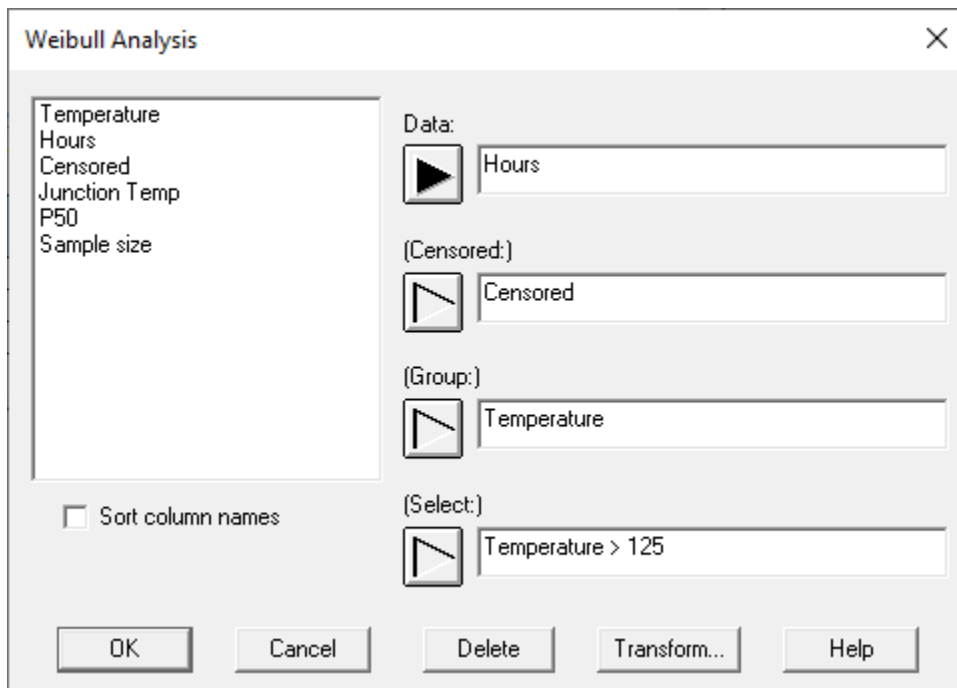
<i>Temperature</i>	<i>Hours</i>	<i>Censored</i>
150	2350	0
150	2560	0
150	2980	0
150	3000	1

150	3000	1
150	3000	1
150	3000	1
150	3000	1
150	3000	1
150	3000	1
175	800	0
175	1130	0
...	...	...

The *Temperature* ranged from 80°C to 200°C. *Hours* is either the time until failure or 3000 if the circuit did not fail. *Censored* equals 0 for an observed failure time or 1 if the unit was still operating after 3000 hours.

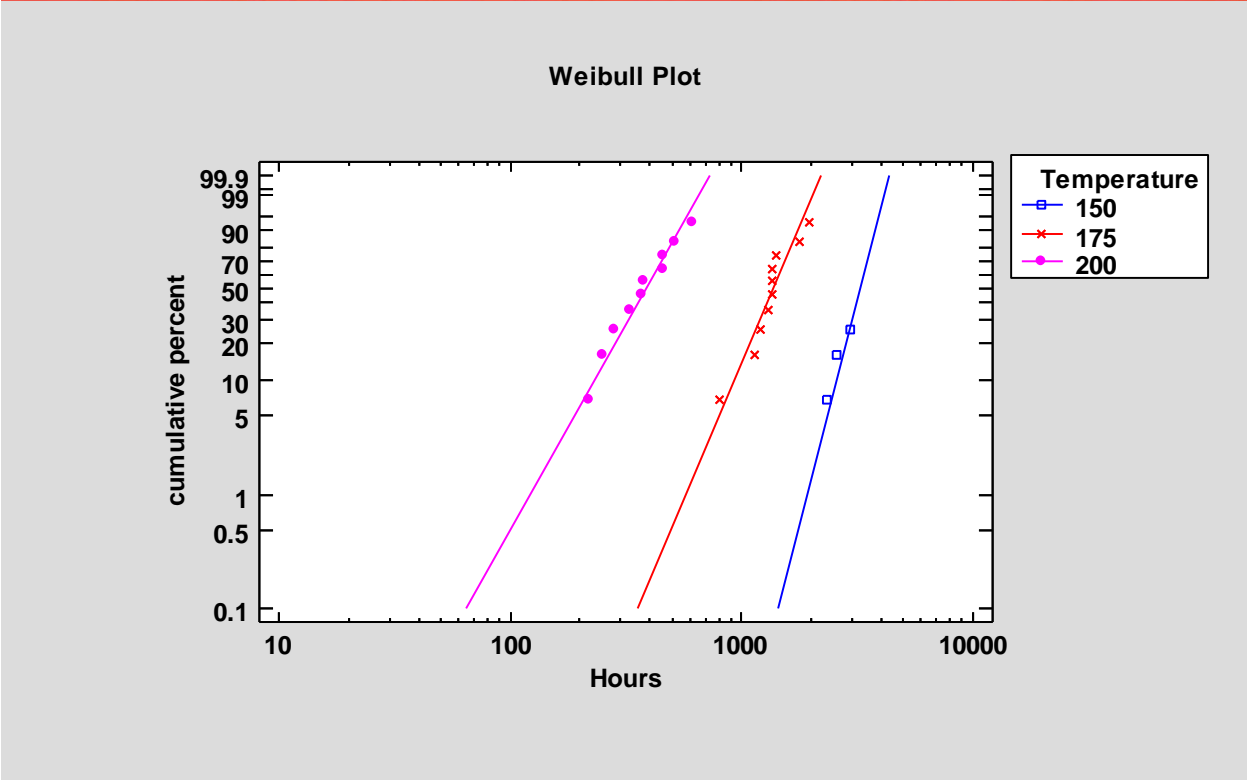
### Estimating Percentiles

In order to create an accelerated life test plot, the MTTF or selected percentile must be estimated for each of the temperatures at which there are observed failures. Assuming a Weibull distribution for the failure times, this can be done using the *Weibull Analysis* procedure. The data input dialog box for that procedure is shown below:



The entry in the *Select* field, *Temperature > 125*, is used to select the temperatures at which there were failures, since no failures occurred at 125°C or below.

The Weibull plot shows that the data are well-described by Weibull distributions:



The *Critical Values* table may be used to estimate the 50<sup>th</sup> percentile at each temperature:

Critical Values for Hours			
Temperature	X	Lower Tail Area (<)	Upper Tail Area (>)
150	3260.62	0.5	0.5
175	1379.61	0.5	0.5
200	388.615	0.5	0.5

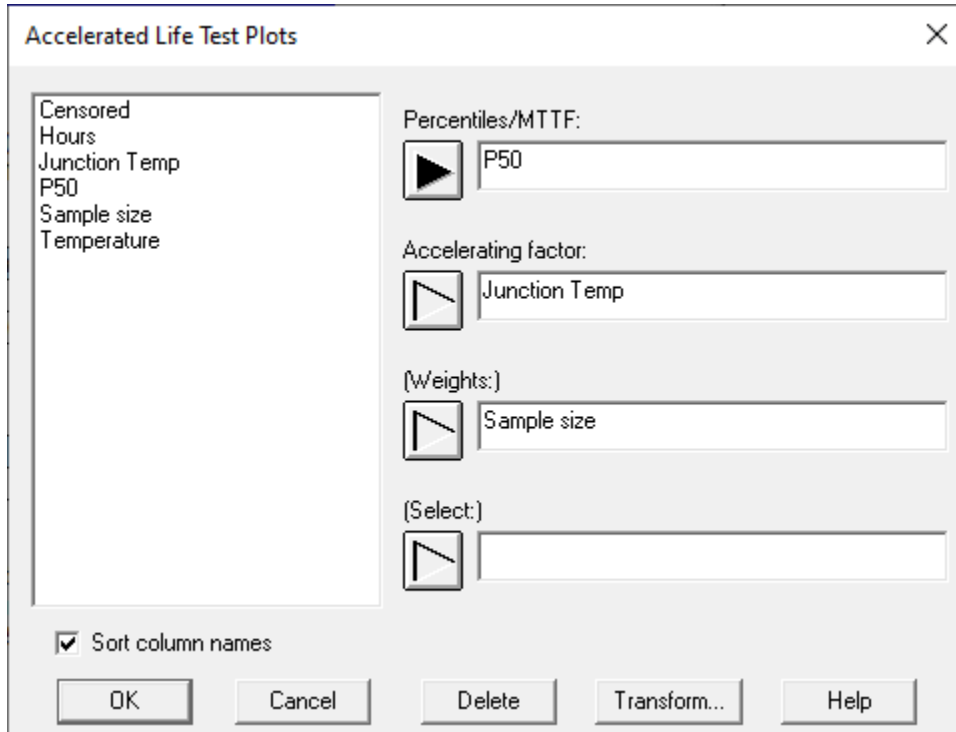
These results have been placed in three additional columns of the *circuits.sgd* file, as shown below:

Junction Temp	P50	Sample Size
150	3260.62	10
175	1379.61	10
200	388.615	10

The above table is the type of data expected by the *Accelerated Life Test Plots* procedure.

## Data Input

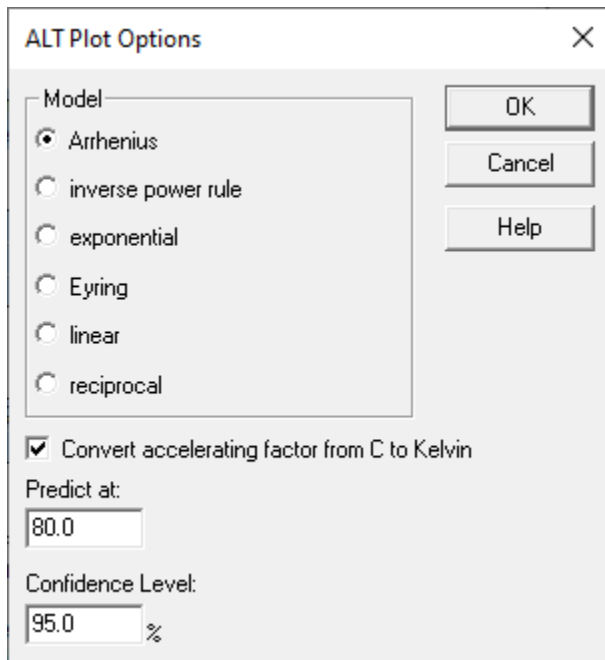
The data input dialog box for the *Accelerated Life Test Plots* procedure expects columns containing percentiles or the MTTF of the failure time distribution and the corresponding temperatures or other accelerating variable:



- **Percentiles/MTTF:** numeric column containing percentiles or the MTTF of the failure time distribution at 2 or more levels of the accelerating factor.
- **Accelerating factor:** numeric column containing the values of the accelerating factor. If the accelerating factor is temperature, it may be entered in either degrees C or degrees Kelvin.
- **Weights:** optional weights to be used when fitting the acceleration model. If the sample sizes corresponding to each level of the accelerating factor are different, they may be used as the weights.
- **Select:** subset selection.

## Analysis Options

The *Analysis Options* dialog box specifies the model to be estimated and the normal operating level at which predictions are desired.



- **Model:** the acceleration model. The available models are:

*Arrhenius model:*

$$Y = A \exp\left(-\frac{E}{kT}\right) \quad (1)$$

where  $T$  is temperature in degrees Kelvin ( $^{\circ}\text{C} + 273.15$ ),  $k = 1/11605$  (Boltzmann's constant), and  $A$  and  $E$  are two unknown parameters. Taking logarithms of both sides:

$$\ln(Y) = \ln(A) - \frac{E}{kT} \quad (2)$$

showing that the log of  $Y$  is linearly related to the reciprocal of  $T$ .

*Inverse power rule model:*

$$Y = \frac{A}{T^n} \quad (3)$$

where  $T$  is the value of the accelerating variable and  $A$  and  $n$  are unknown parameters. The model can be linearized by taking logarithms of both sides:

$$\ln(Y) = \ln(A) - n \ln(T) \quad (4)$$

*Exponential model:*

$$Y = Ae^{-\lambda T} \quad (5)$$

where T is the value of the accelerating variable and A and  $\lambda$  are unknown parameters. The model can again be linearized by taking logarithms of both sides:

$$\ln(Y) = \ln(A) - \lambda T \quad (6)$$

*Eyring model:*

$$Y = \frac{1}{T} e^{-(A-\frac{B}{T})} \quad (7)$$

where T is the value of the accelerating variable and A and B are unknown parameters. It can be linearized as

$$\ln(YT) = -A + \frac{B}{T} \quad (8)$$

*Linear model:*

$$Y = A + BT \quad (9)$$

where T is the value of the accelerating variable and A and B are unknown parameters.

*Reciprocal model:*

$$Y = A + B/T \quad (10)$$

- **Convert accelerating factor from C to Kelvin:** Check this box if the accelerating factor is temperature and it has been entered in degrees C rather than degrees Kelvin.
- **Predict at:** the normal operating level of the accelerating factor. This is the level at which predictions will be made.
- **Confidence Level:** the percentage to be used for the confidence intervals.

## Analysis Summary

The *Analysis Summary* displays statistics for the fitted model and an extrapolation of the model to the specified value of the accelerating factor.

<u>Accelerated Life Test Plots - P50 vs. Junction Temp</u>				
Percentiles: P50				
Temperatures: Junction Temp				
<b>Fitted Arrhenius Model</b>				
P50 = 0.0000070398*exp(0.730577/(k*(Junction Temp+273.15)))				
where k = Boltzmann's constant (8.617E-5 EV/degrees K)				
<b>Regression Statistics</b>				
Coefficient	Estimate	Standard Error	T Statistic	P-Value
Intercept	-11.8639	2.71659	-4.36722	0.1433
Slope	0.730577	0.10458	6.98582	0.0905
R-squared = 97.992%				
<b>Prediction</b>				
Junction Temp	Prediction	Lower 95.0% limit	Upper 95.0% limit	
80.0	187923.	16.8628	2.09425E9	

The table shows:

- **Fitted Model** - the fitted model. In this case, the equation is

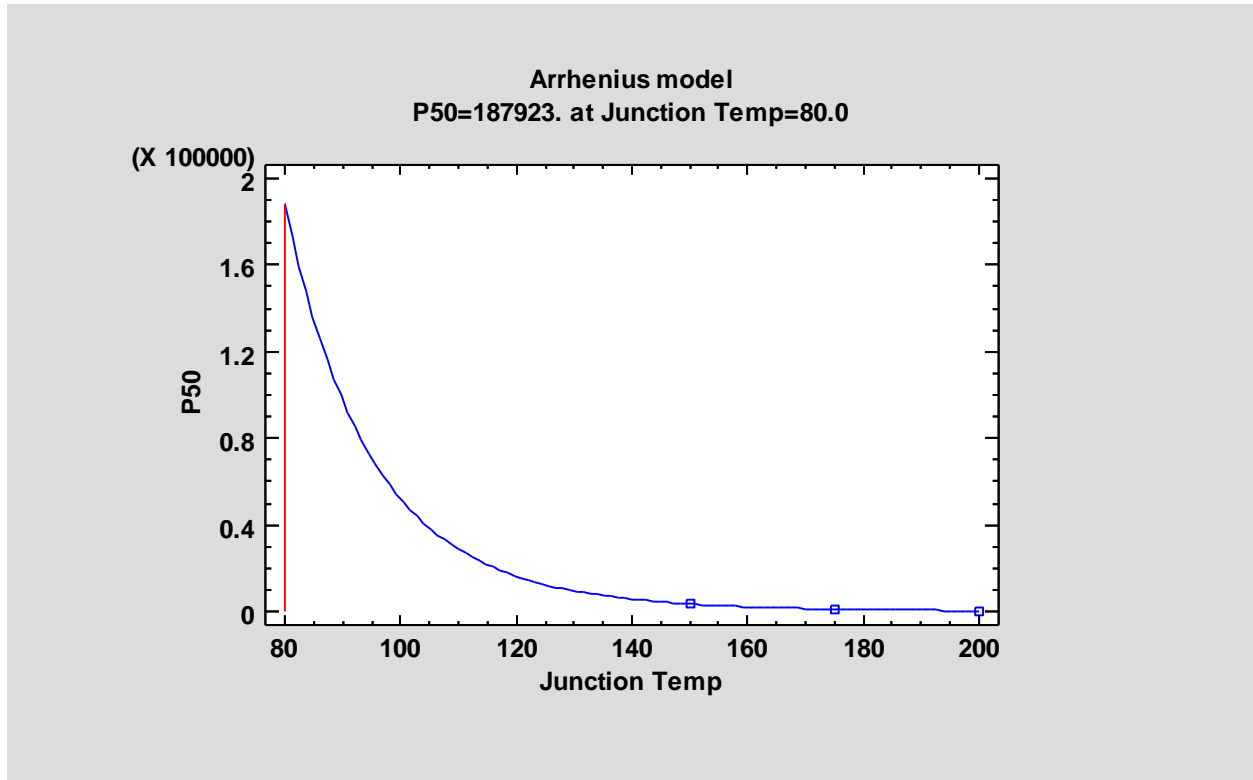
$$P_{50} = 7.04 \times 10^{-6} \left[ \exp \left( \frac{0.7306}{k(T+273.15)} \right) \right] \quad (11)$$

- **Regression Statistics** - the intercept, slope, and R-squared statistic from a linear regression of  $\ln(P_{50})$  against  $1/(k(T+273.15))$ .
- **Prediction** - the estimated percentile at the normal operating temperature, with 95% confidence limits based on the fitted regression model.

In this case, it is estimated that 50% of the items will have failed after 187,923 hours at 80°C.

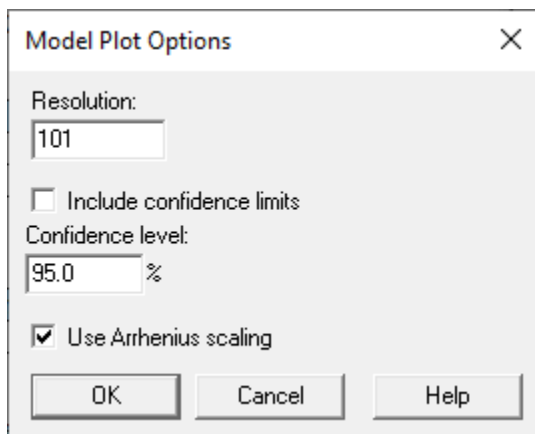
## Fitted Model Plot

The *Fitted Model Plot* shows the data, the fitted model, and the extrapolated percentile.



If the Arrhenius model fits the data well, the points should lie close to the fitted line.

When fitting an Arrhenius model, *Pane Options* may be used to plot the data with scaling specially designed for that model:

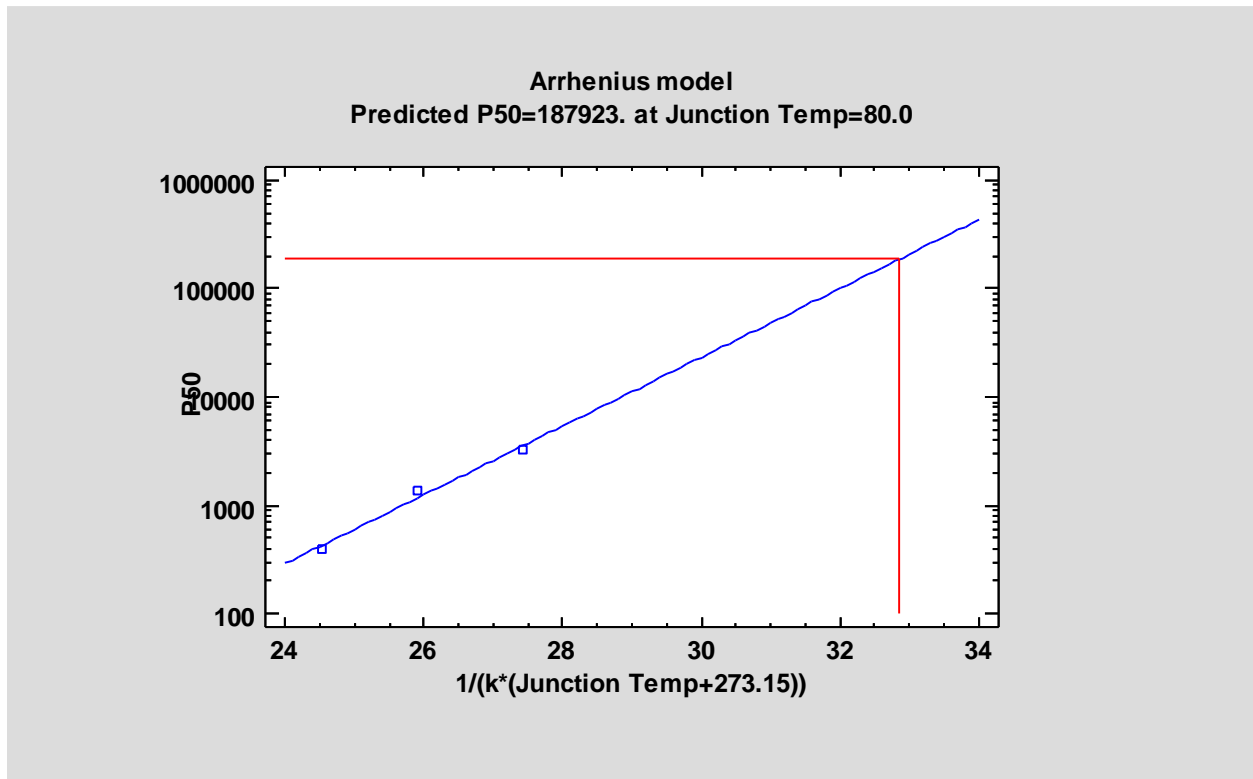


- **Resolution** – number of points at which the fitted model is plotted.



- **Include confidence limits** – whether to include confidence limits for the estimated MTTF or percentile.
- **Confidence level** - level of confidence for the limits.
- **Use Arrhenius scaling** – whether to use scaling specially designed for the Arrhenius model.

Selecting *Use Arrhenius scaling* generates the plot shown below:



With the selected scaling, the model is linear.

## References

Meeker, W. Q. and Escobar, L. A. (1998). Statistical Methods for Reliability Data. New York: John Wiley and Sons.